Regularization by noise for transport and kinetic equations

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Abstract

For some differential equations the addition of a carefully chosen, random noise term can produce a regularizing effect (e.g. solutions are more regular, or restored uniqueness). I will first consider a few easy examples (ODEs) to introduce some of these regularizing mechanisms, then detail two cases where we have regularization for a PDE: the (stochastic) linear transport equation

$$\partial_t u(t,x) + b(t,x)\cdot\nabla u(t,x) \quad \Big(+ \nabla u(t,x) \circ \frac{dW_t}{dt} \Big) = 0$$

and a (stochastic) kinetic equation with force term

$$\partial_t f(t,x,v) + v\cdot\nabla_x f(t,x,v) + F(x)\cdot\nabla_v f(t,x,v) \quad \Big(+ \nabla_v f(t,x,v) \circ \frac{dW_t}{dt} \Big) = 0\,.$$

I will present some classical results for these two equations, related to well-posedness and regularity of solutions, that can be obtained under weaker hypothesis in the stochastic setting.
This work is partially supported by LABEX MILYON / ANR-10-LABX-0070.

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