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# Energy dissipations for non-isothermal models of grain boundary motions

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## Abstract

This study is based on recent jointworks with Prof. Moll, J. S., University of Valencia, Spain.

In this talk, systems of parabolic type PDEs are considered. These are based on phase field models of grain boundary motions, proposed in [Warren, J. A., Kobayashi, R., Lobkovsky, A. E. and Carter, W. C.: *Acta Materialia*, 51 (2003), 6035–6058]. Each of our systems consists of:

(1st.eq.) heat equation,

(2nd.eq.) nonstandard parabolic equation for phase transition,

(3rd.eq.) singular diffusion equation for crystalline orientation,

and in particular, the latter two equations (2nd.eq.)-(3rd.eq.) are derived as a gradient system of a governing energy, called free-energy. Hence, in the light of modelling methods, we can expect that our systems should be to reproduce some energy-dissipations, and the energy-dissipations should bring in some stability for the corresponding dynamical systems.

The aim in this talk is to have certain positive answers for the expectation. To this end, we focus on a special kind of solution, named "energy-dissipative solution", which somehow achieves the energy-dissipation together with the compatibilities with (1st.eq.)-(3rd.eq.).

Under suitable assumptions, several theorems concerned with:

(A) the existence of energy-dissipative solutions,

(B) the large-time behavior for energy-dissipative solutions,

will be demonstrated as the main results of this talk.

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