Singular perturbation problems in perforated domains

Paolo Musolino*1

1Dipartimento di Matematica, Università degli Studi di Padova – Italy

Abstract

The asymptotic behaviour of the solutions of boundary value problems in domains with small holes has been investigated by many authors with different approaches. In this talk, we consider a Dirichlet problem for the Laplace operator in a bounded domain \( \Omega^\epsilon \) of \( \mathbb{R}^n \) containing the origin, where we remove a small set whose size is determined by a parameter \( \epsilon \) and which collapses to 0 for \( \epsilon = 0 \). Then for \( \epsilon \neq 0 \) we denote the solution to such a problem by \( u_\epsilon \). If \( p \in \Omega^\epsilon \) and \( p \neq 0 \), then it makes sense to consider for \( \epsilon \neq 0 \) and ‘small’ the value of the solution \( u_\epsilon \) at the point \( p \). It is natural to ask what can be said on the map which takes \( \epsilon \) small and positive to \( u_\epsilon(p) \) around the degenerate value \( \epsilon = 0 \). One can try to answer to this question in several ways. By the approach proposed by Lanza de Cristoforis, one can show that, if \( n \geq 3 \), then there exist \( \epsilon_\rho > 0 \) and a real analytic function \( U_\rho \) from \( ]-\epsilon_\rho, \epsilon_\rho[ \) to \( \mathbb{R} \) such that \( u_\epsilon(p) = U_\rho(\epsilon) \) for all \( \epsilon \in ]0, \epsilon_\rho[ \), and one can then investigate the validity of such an equality for \( \epsilon \) negative. After an introductory part on the case of dimension \( n \geq 3 \), we will turn to consider the two-dimensional case.

Based on joint works with M. Dalla Riva (CIDMA, Universidade de Aveiro) and S.V. Rogosin (Belarusian State University).

*Speaker