Some Qualitative Properties of a Fractional-type Camassa-Holm Equation

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Abstract

The equation of motion of one-dimensional nonlocal nonlinear elasticity is:

$$u_{tt} = (\beta * (u + g(u)))_{xx}$$

where $*$ denotes the spatial convolution operator and $\beta(x)$ is the kernel function. In a recent work [1], using asymptotic expansion method, a fractional generalization of the Camassa-Holm equation have been derived by showing that, when the Fourier transform of the kernel function $\beta(x)$ has fractional powers, the unidirectional propagation of small-but-finite amplitude waves are governed by the fractional Camassa-Holm equation:

$$u_t + \frac{6}{5}u_x + 3uu_x + (-D_x^2)^\nu u_t = -\frac{3}{5}[2(-D_x^2)^\nu(uu_x) + u(-D_x^2)^\nu u_x] \tag{1}$$

with a constant $\nu \geq 1$ being not necessarily an integer and $(-D_x^2)^\nu \omega = \mathcal{F}^{-1}(|\xi|^{2\nu}\mathcal{F}\omega)$ where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse, respectively.
In this talk, we discuss local well-posedness of solutions to the Cauchy initial value problem for (1) and wave breaking phenomenon.

References

[1] H. A. Erbay, S. Erbay, A. Erkip *Derivation of the Camassa-Holm equations for elastic waves*, Physics Letters A , **379**, 2015, 956-961.

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