A critical problem for fractional Laplacians in contractible domains

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Abstract

We study the problem

$$\begin{cases} (-\Delta)^s u = u^{\frac{N+2s}{N-2s}} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega \end{cases}$$

involving the fractional Laplacian $(-\Delta)^s$. Here, $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ is a bounded domain with smooth boundary,

$$(-\Delta)^s u(x) = C(N,s) \lim_{\varepsilon \to 0} \int_{\complement B_\varepsilon(x)} \frac{u(x)-u(y)}{|x-y|^{N+2s}}\, dy \quad \text{for } u \in C^\infty(\mathbb{R}^N) \cap \dot{H}^s(\mathbb{R}^N),$$

where $s \in (0,1)$ and $C(N,s)$ is a positive constant. Recently, such kind of problems are studied by many researchers. Ros-Oton and Serra [ARMA 213 (2014)] showed if Ω is star-shaped, then the problem does not admit solutions. Secchi, Squassina and the speaker [DIE 28 (2015)] showed that if Ω has a small hole then the problem has a solution. Servadei and Valdinoci [TAMS 365 (2015)] studied the Brezis-Nirenberg problem, which is closely related to the problem. In this talk, we will show that if $N \geq 3$ and $s \in (0,1)$, or $N = 2$ and $s \in (0, 1/2]$, then there is a contractible domain Ω such that the problem has a solution. We will give such a domain. This result is based on a joint work with Mosconi and Squassina.

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