On some new properties of the solution to an initial-boundary-value problem for the time-fractional diffusion equation with $\alpha \in (0, 2)$

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Abstract

The time-fractional diffusion equation is a fractional partial differential equation which interpolates the heat equation and the wave equation via fractional derivatives in Caputo's sence and which provides a mathematical model for the intermediate behaviour of linear viscoelastic media subject to stress pulses.

In this work we study the following initial-boundary-value problem in the upper half-plane for the one-dimensional time fractional diffusion equation with $\alpha \in (0, 2)$:

$$\begin{cases} \frac{\partial^\alpha}{\partial t^\alpha} u(x,t) = \lambda \frac{\partial^2}{\partial x^2} u(x,t) & \text{if } x \in \mathbb{R}; \quad t \in \mathbb{R}_0^+ \\ u(x,0) = \varphi(x) & \text{if } x \in \mathbb{R} \\ u \text{ bounded} \end{cases}$$

A solution to this problem, identical to the one given by other authors, is obtained; as well as the first proof to this result. Different new properties of the solution to this problem, which are strongly connected to the physical context by viscoelastic media, are presented. Among these properties, the asymtotic behaviour and the limiting behaviour $\alpha \to 1$ and $\alpha \to 2^-$ stand out.

Finally, some new advances considering the generalization of the order of derivation $\alpha > 0$ are presented.

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