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# Analysis of degenerate cross-diffusion population models with volume filling

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## Abstract

A class of parabolic cross-diffusion systems modeling the interaction of an arbitrary number of population species is analyzed in a bounded domain with no-flux boundary conditions. The equations are formally derived from a random-walk lattice model in the diffusion limit. Compared to previous results in the literature, the novelty is the combination of general degenerate diffusion and volume-filling effects. Conditions on the nonlinear diffusion coefficients are identified, which yield a formal gradient-flow or entropy structure. This structure allows for the proof of global-in-time existence of bounded weak solutions and the exponential convergence of the solutions to the constant steady state. The existence proof is based on an approximation argument, the entropy inequality, and new nonlinear Aubin-Lions compactness lemmas. The proof of the large-time behavior employs the entropy estimate and convex Sobolev inequalities. Moreover, under simplifying assumptions on the nonlinearities, the uniqueness of weak solutions is shown by using the  $H^{-1}$  method, the  $E$ -monotonicity technique of Gajewski, and the subadditivity of the Fisher information.

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