
On the wave length of smooth periodic traveling waves of the Camassa-Holm equation

Anna Geyer*¹ and Jordi Villadelprat

¹Universitat Autònoma de Barcelona (UAB) – 08193 Bellaterra (Barcelona) SPAIN, Spain

Abstract

This paper is concerned with the wave length λ of smooth periodic traveling wave solutions of the Camassa-Holm equation. The set of these solutions can be parametrized using the wave height a (or “peak-to-peak amplitude”). Our main result establishes monotonicity properties of the map $a \mapsto \lambda(a)$, i.e., the wave length as a function of the wave height. We obtain the explicit bifurcation values, in terms of the parameters associated to the equation, which distinguish between the two possible qualitative behaviours of $\lambda(a)$, namely monotonicity and unimodality. The key point is to relate $\lambda(a)$ to the period function of a planar differential system with a quadratic-like first integral, and to apply a criterion which bounds the number of critical periods for this type of systems.

*Speaker