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# Existence of solitary-wave solutions to nonlocal equations

Mathias Nikolai Arnesen\*<sup>1</sup>

<sup>1</sup>Department of Mathematical Sciences, Norwegian University of Science and Technology (NTNU) –  
7491 Trondheim, Norway

## Abstract

We prove existence and conditional energetic stability of solitary wave solutions for the two classes of pseudodifferential equations  $u_t + (f(u))_x - (Lu)_x = 0$  and  $u_t + (f(u))_x + (Lu)_t = 0$ , where  $f$  is a nonlinear term, typically of the form  $c|u|^p$  or  $cu|u|^{p-1}$ , and  $L$  is a Fourier multiplier operator of positive order. The former class includes for instance the Whitham equation with capillary effects and the generalized Korteweg–de Vries equation, and the latter the Benjamin–Bona–Mahony equation. Existence and conditional energetic stability results have earlier been established using the method of concentration–compactness for a class of operators with symbol of order  $s \geq 1$ . We extend these results to symbols of order  $0 < s < 1$ , thereby improving upon the results for general operators with symbol of order  $s \geq 1$  by enlarging both the class of linear operators and nonlinearities admitting existence of solitary waves. Instead of using abstract operator theory, the new results are obtained by direct calculations involving the non-local operator  $L$ , something that gives us the bounds and estimates needed for the method of concentration–compactness.

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\*Speaker