
Perihelia reduction in the planetary problem

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Abstract

In the XIX century, Jacobi discovered that the motions of three bodies in gravitational interaction may be described by a system of the eighth order, rather than eighteenth. Some degrees of freedom might be neglected by the invariance of the problem by translations and rotations. After him, Radau wrote these equations in Hamiltonian form: he introduced a Hamilton function and four couples of canonical coordinates. The reduction of order by Jacobi and Radau has been extensively applied in the literature, and, for about one century and one half, it appeared as the only available one. In 1963, V.I. Arnold wrote that the lack of a generalization of it to the case of more bodies was an obstacle to the extension of his theorem of stability of planetary motions. In 1982-1983 F. Boigey and A. Deprit extended Jacobi-Radau's reduction to the general case. The coordinates by Boigey and Deprit were next rediscovered by the author and applied to the problem [Chierchia and Pinzari, 2011], allowing for a direct proof of Arnold's statement. Important feature of JRBD reduction are: (i) they are not defined for the problem constrained in the plane and (ii) no symmetry in the Hamiltonian appears, relatively to the invariance of the problem by reflections. We present an alternative reduction based on the perihelia of instantaneous orbits that takes into account items (i) and (ii). Next, we shall show how these items allow for a more global formulation of Arnold's statement.

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