Some weighted estimates for elliptic operators with singular data

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Abstract

Let Ω be an open subset of $\mathbb{R}^n$, $n \geq 2$, not necessarily bounded or regular. Consider in Ω the elliptic operator

$$L = \sum_{i,j=1}^{n} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} a_i \frac{\partial}{\partial x_i} + a \,,$$

where the coefficients a_{ij} are locally VMO while the data a_i, a belong to a weighted Sobolev space whose weight σ is a function of distance type from a nonempty subset of $\partial\Omega$. We prove that if $s \in \mathbb{R}$, $p \in]n/2, +\infty[$ and u is a solution of the problem

$$\begin{cases} u \in W^{2,p}_{loc}(\Omega) \\ Lu \geq f \quad f \in L^p_{loc}(\Omega) \\ \limsup\limits_{x \to x_o} \sigma^s(x)\, u(x) \leq 0 \qquad \forall\, x_o \in \partial\Omega \\ \limsup\limits_{|x| \to +\infty} \sigma^s(x)\, u(x) \leq 0 \quad \text{if } \Omega \text{ is unbounded,} \end{cases}$$

then u verifies the following Aleksandrov type estimate

$$\sup_{x \in \Omega} \sigma^s(x) u(x) \leq c \Big(\frac{1}{|B|} \int_B |\sigma^{s+2} f^-|^p \, dx \Big)^{\frac{1}{p}} ,$$

where $B \subset\subset \Omega$ is an open ball, f^- is the negative part of f and $c \in \mathbb{R}_+$ is independent of u. As a consequence, some uniqueness results for singular elliptic problems are obtained.

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