Symmetric mountain pass lemma and sublinear elliptic equations

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Abstract

We study the p-Laplace elliptic equation

$$-\Delta_p u = f(x, u) \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian and Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary. We assume that $f(x, u)$ is a continuous function which is odd with respect to u, i.e., $f(x, -u) = -f(x, u)$. Then equation (1) satisfies either (A) or (I) below:
(A) the zero solution is an accumulation point of the set of all solutions,
(I) the zero solution is an isolated point of the set of all solutions.
Applying the symmetric mountain pass lemma in our paper [Kajikiya, J. Funct. Anal., **225**, (2005) 352–370], we give a weak sufficient condition for (A). Moreover, we decide which type of (A) or (I) holds for some elliptic equations.

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