Breaking of Ergodicity in Expanding Systems of Globally Coupled Piecewise Affine Circle Maps

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Abstract

To identify and to explain coupling-induced phase transitions in Coupled Map Lattices (CML) has been a lingering enigma for about two decades. In numerical simulations, this phenomenon has always been observed preceded by a lowering of the Lyapunov dimension, suggesting that the transition might require changes of linear stability. Yet, recent proofs of co-existence of several phases in specially designed models work in the expanding regime where all Lyapunov exponents remain positive.

In this talk, I will consider a family of CML composed by piecewise expanding individual maps, global interaction and finite number N of sites, in the weak coupling regime where the CML is uniformly expanding.

I will show, mathematically for N=3 and numerically for N¿3, that a transition in the asymptotic dynamics occurs as the coupling strength increases. The transition breaks the (Milnor) attractor into several chaotic pieces of positive Lebesgue measure, with distinct empiric averages. It goes along with various symmetry breaking, quantified by means of magnetization-type characteristics.

Despite that it only addresses finite-dimensional systems, to some extent, this result reconciles the previous ones as it shows that loss of ergodicity/symmetry breaking can occur in basic CML, independently of any decay in the Lyapunov dimension.

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