# A Mathematical Justification of the Thin Film Approximation for the Flow down an Inclined Plane

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**Abstract**

We consider a two-dimensional motion of a thin film flowing down an inclined plane under the influence of the gravity and the surface tension. The motion is mathematically formulated as a free boundary problem for the incompressible Navier–Stokes equations. It is hard to analyze the Navier–Stokes equations directly in order to investigate the stability and the instability of the surface waves, so that we often use a thin film approximation. It is an approximation obtained by the perturbation expansion with respect to the aspect ratio $\delta$ of the film under the thin film regime $\delta \ll 1$. The famous examples of the approximate equations are the Burgers equation, the Kuramoto–Sivashinsky equation, the KdV–Burgers equation, the KdV–Kuramoto–Sivashinsky equation, the Benney equation, and so on. We give a mathematically rigorous justification of a thin film approximation by establishing an error estimate between the solution of the Navier–Stokes equations and those of approximate equations under the assumptions that the Reynolds number, the angle of inclination, and initial data are sufficiently small.

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