Multi-scale Dynamics in Microstructures

Annalisa Iuorio*[1], Christian Kühn[1], and Peter Szmolyan[1]

[1]Institute for Analysis and Scientific Computing, Vienna University of Technology – Austria

Abstract

We consider a singularly perturbed, non-convex variational problem describing microstructures in one space dimension (in particular, simple laminates). The minimizers of the functional exhibit a complicated multi-scale behavior. We propose a dynamical systems approach to analyse the critical points of the functional by applying methods from geometric singular perturbation theory to the corresponding Euler-Lagrange equation. Some microstructures can be interpreted as periodic solutions of the system obtained by writing the Euler-Lagrange equation as a 4-dimensional singularly perturbed Hamiltonian system of ODEs depending on a small parameter $0 < \varepsilon \ll 1$. Starting from a singular orbit ($\varepsilon = 0$), we prove for $\varepsilon \neq 0$ the existence of a family of periodic orbits parametrized by the value μ of the Hamiltonian for ε small. The periodicity of the solutions explains the observed multi-scale structures. Numerical computations based on AUTO are performed in order to study quantitative properties and bifurcations of solutions as key parameters ε and μ vary.

*Speaker