Pinning and snaking on lattices: orientation dependence and multiple pinning mechanisms

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Abstract

We study pinning of fronts and the associated homoclinic snaking on bistable lattices. Bistability leads to the existence of travelling waves comprising a front connecting the two stable states. In the analogous continuous system, the wavespeed varies smoothly and is zero at a single point only. In contrast, a lattice system exhibits a range of values at which the front is stationary, due to the energy barrier imposed by the periodic lattice. Within this range, back-to-back fronts may be glued together forming localised solutions which 'snake' back and forth in parameter space via successive saddle-node bifurcations. We first look at one-dimensional fronts on a planar lattice, and employ the method of exponential asymptotics in the continuum limit to derive the width of the pinning region and the associated snaking bifurcation diagram. In particular, we show that the width depends discontinuously on the orientation ϕ of the front to the underlying lattice, vanishing when $\tan\phi$ is irrational, and is exponentially small otherwise. Standard asymptotic techniques based solely on algebraically small quantities cannot derive such an effect. We then consider a fourth-order, pattern-forming lattice problem in one dimension. In this case there are two periodic structures to which a front may pin: the lattice and the sinusoidal pattern. We investigate how including a third spatial scale in this manner impacts the the snaking bifurcation diagram.

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