Asymptotic behavior of interfaced solutions to parabolic equations

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Abstract

We study the asymptotic behaviour of solutions to parabolic equations in one dimensional bounded domains, that is

$$\partial_t u = \mathcal{F}^\varepsilon[u], \quad u(x,0) = u_0(x) \tag{1}$$

where $\mathcal{F}^\varepsilon$ denotes a nonlinear parabolic differential operator. We emphasis
on the phenomenon known as **metastable dynamics**, whereby the time-dependent solution approaches its steady
state in an asymptotically exponentially long time interval as the viscosity coefficient ε goes to zero. To study such behavior, we propose a general framework based on choosing a family of approximate steady states $\{U^\varepsilon(\cdot;\xi)\}_{\xi \in J}$
and on the spectral properties of the linearized operators at such states. The parameter ξ usually represents the location of the internal interface, so that the slow motion of solutions $u = U^\varepsilon + v$
is analyzed by means of an ODE for the parameter $\xi = \xi(t)$, coupled with a PDE
for the perturbation $v := u - U^\varepsilon(\cdot;\xi)$.
We then apply the general strategy to some explicit examples, such as viscous conservation laws with a reaction term and reaction diffusion equations. We will see how the speed rate of convergence of the solution towards the stable configuration of the system is influenced by the stability properties of the metastable steady state and by the explicit form of the nonlinear terms.

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