Multiplicity results for sign changing bound state solutions of a semilinear equation

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Abstract

In this talk we give conditions on the nonlineary f so that the problem

$$\Delta u + f(u) = 0, \quad x \in \mathbb{R}^N, N \geq 2, \quad \lim_{|x|\to\infty} u(x) = 0,$$

has at least two solutions having a prescribed number of nodal regions and for which $u(0) > 0$. Any nonconstant solution to (1) is called a bound state solution. Bound state solutions such that $u(x) > 0$ for all $x \in \mathbb{R}^N$, are referred to as a first bound state solution, or a ground state solution. The existence of ground states for (1) has been established by many authors under different regularity and growth assumptions on the nonlinearity f, both for the Laplacian operator and the degenerate Laplacian operator. The main assumptions on the nonlinearity f are

(f_1) f is a continuous function defined in $\mathbb{R}$, and f is locally Lipschitz in $\mathbb{R} \setminus \{0\}$.

(f_2) There exists $\delta > 0$ such that if we set $F(s) = \int_0^s f(t)dt$, it holds that $F(s) < 0$ for all $0 < |s| < \delta$, and $\lim_{s\to-\infty} F(s) = \lim_{s\to\infty} F(s)$, $F(s) < \lim_{s\to\infty} F(s)$ for all $s \in \mathbb{R}$.

(f_3) F has a local maximum at some $\gamma \in (\delta, \infty)$ and $F(\gamma) > 0$.

(f_4) there exists $\theta \in (0, 1)$ such that

$$\lim_{s\to\infty} \Big(\inf_{s_1,s_2\in[\theta s,s]} Q(s_2) \Big(\frac{s}{f(s_1)}\Big)^{N/2} \Big) = \infty, \qquad (1)$$

where $Q(s) := 2NF(s) - (N-2)sf(s)$.

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