Delay evolution equations with measures and nonlocal initial conditions

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Abstract

We consider the semilinear delay evolution equation with measures and nonlocal initial data of the form

\[
\begin{aligned}
  du(t) &= \{Au(t) + f(t, u_t)\}dt + dg(t), \quad t \in \mathbb{R}_+,
  \\
  u(t) &= h(u)(t), \quad t \in [-\tau, 0],
\end{aligned}
\]

where $\tau \geq 0$, $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a $C_0$-semigroup, $f : \mathbb{R}_+ \times \mathcal{R}([-\tau, 0]; X) \to X$ is continuous, $g \in BV_{\text{loc}}(\mathbb{R}_+; X)$ and $h : \mathcal{R}_b(\mathbb{R}_+; X) \to \mathcal{R}([-\tau, 0]; X)$ is nonexpansive, and we prove an existence result for $L^\infty$-solutions. Here $\mathcal{R}_b(\mathbb{R}_+; X)$ stands for the space of bounded and piecewise continuous functions from $\mathbb{R}_+$ to $X$ and $\mathcal{R}([-\tau, 0]; X)$ denotes the space of piecewise continuous functions from $\mathbb{R}_+$ to $X$, both endowed with the sup-norm.

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