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# Delay evolution equations with measures and nonlocal initial conditions

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## Abstract

We consider the semilinear delay evolution equation with measures and nonlocal initial data of the form

$$\begin{cases} du(t) = \{Au(t) + f(t, u_t)\}dt + dg(t), & t \in \mathbb{R}_+, \\ u(t) = h(u)(t), & t \in [-\tau, 0], \end{cases}$$

where  $\tau \geq 0$ ,  $A : D(A) \subseteq X \rightarrow X$  is the infinitesimal generator of a  $C_0$ -semigroup,  $f : \mathbb{R}_+ \times \mathcal{R}([-\tau, 0]; X) \rightarrow X$  is continuous,  $g \in BV_{\text{loc}}(\mathbb{R}_+; X)$  and  $h : \mathcal{R}_b(\mathbb{R}_+; X) \rightarrow \mathcal{R}([-\tau, 0]; X)$  is nonexpansive, and we prove an existence result for  $\mathcal{L}^\infty$ -solutions. Here  $\mathcal{R}_b(\mathbb{R}_+; X)$  stands for the space of bounded and piecewise continuous functions from  $\mathbb{R}_+$  to  $X$  and  $\mathcal{R}([-\tau, 0]; X)$  denotes the space of piecewise continuous functions from  $\mathbb{R}_+$  to  $X$ , both endowed with the sup-norm.

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