Nonlinear evolution equations arising from mathematical biology and medicine

Akisato Kubo

School of Health Sciences, Fujita Health University (FHU) – 1-98 Dengakugakubo,Kutsukake-cho,Toyoake,Aichi.470-1192, Japan

Abstract

In this talk we consider initial-Neumann boundary value problem of nonlinear evolution equations with strong dissipation and proliferation arising from mathematical biology and medicine formulated as

\[(NE) \begin{align*}
  u_{tt} &= D \nabla^2 u_t + \nabla \cdot (\chi(u_t, e^{-u})e^{-u}\nabla u) + \mu_1 u_t(1 - u_t) \quad \text{in } (x, t) \in \Omega \times (0, \infty) \\
  \frac{\partial}{\partial \nu} u|_{\partial \Omega} &= 0 \quad \text{on } \partial \Omega \times (0, \infty) \\
  u(x, 0) &= u_0(x), u_t(x, 0) = u_1(x) \quad \text{in } \Omega
\end{align*} \]

where constants \(D, \mu_1\) are positive, \(\Omega\) is a bounded domain in \(\mathbb{R}^n\) with a smooth boundary \(\partial \Omega\) and \(\nu\) is the outer unit normal vector. Under some regularity and boundedness conditions of the coefficient \(\chi(u_t, e^{-u})\) of \((1.1)\), we derive the energy estimate of \((NE)\), which enables us to show the global existence in time and asymptotic behavior of the solution. We deal with mathematical models of tumor migration as an extended case of \((NE)\) and applying our result to them we discuss the behavior of cell migration.