
Recent results on the Kuramoto–Sakaguchi equation

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Abstract

The Kuramoto-Sakaguchi (or just Kuramoto) equation is a Fokker-Planck equation, where the space variable is an angle, and the drift is quadratically nonlinear through an integral which accounts for the action of the infinitely many oscillators on any given one of them. This PDE provides a model which describes a variety of phenomena, in particular self-synchronization of chemical and biological oscillations, hence in neurosciences, as well as in physical and social systems. An additional integration over all frequencies appears, which makes the model even more nonstandard from the mathematical viewpoint.

It is the parabolic equation with non-standard non-linear integral term. Integration is calculated with respect to the problem coefficient over unbounded interval. The results available in the literature concerning parabolic equations, or even integro-parabolic equations, cannot be applied to this case. The Kuramoto-Sakaguchi equation is here considered when the “frequency distribution”, the frequency being an independent variable in the model equation, has an *unbounded* support. This case, interesting in view of applications, have not been considered earlier.

Problem solvability is established.

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