Global existence in a 2D semilinear chemotaxis-Navier-Stokes system with position sensitivity under the small initial data

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Abstract

We will prove that a coupled semilinar chemotaxis-fluid model with position dependent sensitivity has a global solution under suitable smallness conditions on the initial data.

We deal with the system

$$\begin{cases} n_t = \Delta n - \nabla \cdot (S(n,c,x)n\nabla c) - u \cdot \nabla n, \\ c_t = \Delta c - nc - u \cdot \nabla c, \\ u_t = \Delta u - (u \cdot \nabla)u - \nabla P + n\nabla\phi, \\ \nabla \cdot u = 0, \end{cases} \tag{KSNS}$$

under the no-flux boundary conditions for n, c and the Dirichlet boundary conditions for u in a bounded domain $\Omega \subset \mathbb{R}^2$. $\phi \in W^{1,\infty}(\Omega)$ is a known function. We assume that the matrix valued position sensitivity $S(n,c,x) = (s_{ij}(n,c,x))_{i,j\in\{1,2\}}$ satisfies $|S(n,c,x)| \le \tilde{S}(c)$, where $\tilde{S}$ is a non-decreasing function on $[0,\infty)$.

When $S = I$ (identity matrix), Winkler (2012, 2014) proved global existence and stabilization in a semilinear system, moreover, Francesco-Lorz-Markowich (2010) obtained that degenerate systems (namely, Δn is replaced with Δn^m $(m > 1)$) have global solutions when $m \in (\frac{3}{2}, 2]$. In contrast, when $S \neq I$ Ishida (2015) found global bounded solutions of degenerate systems with $m \in (1,\infty)$. However, she used the condition $m > 1$, so global solvability of the semilinear system with $S \neq I$ is open. We are going to give an answer to it.

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