Global existence and boundedness in a quasilinear degenerate Keller-Segel system

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Abstract

We study global existence and boundedness of weak solutions to the quasilinear degenerate Keller–Segel system

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\begin{align*}
    u_t & = \nabla \cdot (\nabla u^m - u^{q-1} \nabla v) \quad \text{in } \mathbb{R}^N \times (0, \infty), \\
v_t & = \Delta v - v + u \quad \text{in } \mathbb{R}^N \times (0, \infty), \\
u(x,0) & = u_0(x), \quad v(x,0) = v_0(x), \quad x \in \mathbb{R}^N,
\end{align*}
\]

where $N \in \mathbb{N}$, $m \geq 1$, $q \geq 2$ and the initial data $(u_0, v_0)$ is assumed to be a pair of non-negative functions. Global existence of weak solutions to (KS) was first established under the condition $q \leq m$ by Sugiyama and Kunii [2]. The condition for $q$ was relaxed into the condition $q < m + 2/N$ by a joint work [1] with Sachiko Ishida. Both results gave only global existence and it is still open whether (KS) admits a weak solution that is uniformly-in-time bounded. We would like to give an answer to this open question.

References
