Fractional operators with singular drift: Smoothing properties and Morrey-Campanato spaces

Diego Chamorro*[1] and Stéphane Menozzi[1]

[1]Laboratoire de Mathématiques et Modélisation d'Evry UMR CNRS 8071 (LaMME) – Université d'Evry-Val d'Essonne – 23 Boulevard de France 91037 Evry Cedex, France

Abstract

We investigate some smoothness properties for the following equation involving a class of Lévy type operators with singular divergence-free drift:

$$\begin{cases} \partial_t \theta(t,x) + \nabla \cdot (v\,\theta)(t,x) + \mathcal{L}\theta(t,x) = 0, \\ \theta(0,x) = \theta_0(x), \qquad \text{for } x \in \mathbb{R}^n,\ n \geq 2, \\ \text{with } \nabla \cdot (v) = 0 \text{ and } t \in [0,T]. \end{cases}$$

The operator $\mathcal{L}$ is given by the expression $\mathcal{L}(f)(x) = \text{v.p.} \int_{\mathbb{R}^n} \big[f(x) - f(x-y)\big]\pi(y)dy$, where $\pi(y)dy$ is a Lévy measure.

The underlying motivation of this framework is given by equations from fluid dynamics. Our argument is based on a duality method using the molecular decomposition of Hardy spaces through which we derive some Hölder continuity. This property will be fulfilled as far as the drift v belongs to a suitable Morrey-Campanato space $M^{q,a}$ for which the regularizing properties of the Lévy operator suffice to obtain global Hölder continuity.

*Speaker