Preventing blow-up in the 2D chemotaxis system by decaying signal-dependent sensitivity

Kentarou Fujie*1 and Takasi Senba2

1Tokyo University of Science – Tokyo, 162-8601, Japan
2Kyushu Institute of Technology – Fukuoka, 804-8550, Japan

Abstract

We will report latest development in a Keller-Segel system with decaying signal-dependent sensitivity. More precisely, we establish absence of blow-up in the Keller-Segel system

\[
\begin{align*}
\frac{\partial u}{\partial t} &= \Delta u - \nabla \cdot (u \nabla \chi(v)) \quad \text{in } \Omega \times (0, \infty), \\
\tau \frac{\partial v}{\partial t} &= \Delta v - v + u \quad \text{in } \Omega \times (0, \infty),
\end{align*}
\]

under the Neumann boundary condition in a smoothly bounded domain in 2D with general sensitivity function \(\chi(v)\) satisfying \(\chi'(s) > 0\) and decaying property: \(\chi'(s) \to 0\) as \(s \to \infty\). As to the case of the linear sensitivity function \(\chi(v) = v\), it is well known that the large initial data induces blow-up solutions. In contrast, the logarithmic case \(\chi(v) = \chi_0 \log v (\chi_0 > 0)\) enables us to see an absolutely different picture from linear one. Namely, independently the size of initial data, global existence and boundedness in the system were proved under some smallness condition on \(\chi_0\) by Nagai-Senba (1998), Biler (1999), Winkler (2011) etc. In this context, removing the smallness condition on \(\chi_0\) in 2D was conjectured by Biler and Velázquez. We give an answer to this conjecture in a more general setting. Moreover, we note that unlike the previous results, our method does not depend on any particular structure of \(\chi(v)\). The main idea is focusing on concentration of mass around each point of \(\Omega\) locally.