Optimal regularity and long-time behavior of solutions for the Blackstock-Crighton equation

Rainer Brunnhuber∗ and Stefan Meyer

1Universität Klagenfurt – Universitätsstraße 65-67, 9020 Klagenfurt, Austria
2Martin Luther Universität Halle-Wittenberg – Theodor-Lieser-Straße 5, 06120 Halle (Saale), Germany

Abstract

We are going to consider the non-homogeneous Dirichlet boundary value problem

\[
\begin{aligned}
\left( a \Delta - \partial_t \right) \left( u_{tt} - b \Delta u_t - c^2 \Delta u \right) &= (k(u_t)^2 + |\nabla u|^2)_{tt} \quad \text{in } \mathbb{R}^+ \times \Omega, \\
(u, u_t, u_{tt}) &= (u_0, u_1, u_2) \quad \text{on } \{t = 0\} \times \Omega, \\
(u, \Delta u) &= (g, h) \quad \text{on } \mathbb{R}^+ \times \Gamma,
\end{aligned}
\]

which is motivated from the Blackstock–Crighton equation modeling the propagation of finite-amplitude sound in thermoviscous fluids. The spatial domain \( \Omega \subset \mathbb{R}^n \) is assumed to have smooth boundary \( \Gamma \). Moreover, \( u_0, u_1, u_2: \Omega \to \mathbb{R} \) and \( g, h: J \times \Gamma \to \mathbb{R} \) are given, \( u = u(t, x) \) is the unknown, and \( a, b, c, k \) are positive constants.

We show that, for small initial and boundary data, there exists a unique global solution with optimal \( L^p \)-regularity which converges to zero at an exponential rate as time tends to infinity. Our techniques are based on maximal \( L^p \)-regularity for parabolic problems and the implicit function theorem.

Additionally, we will provide a short outlook on how to treat the case of non-homogeneous Neumann boundary conditions which are relevant for applications of high intensity focused ultrasound in a medical context.

∗Speaker