Optimal regularity and long-time behavior of solutions for the Blackstock-Crighton equation

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Abstract

We are going to consider the non-homogeneous Dirichlet boundary value problem

$$\begin{cases} (a\Delta - \partial_t)(u_{tt} - b\Delta u_t - c^2\Delta u) = (k(u_t)^2 + |\nabla u|^2)_{tt} & \text{in } \mathbb{R}_+ \times \Omega, \\ (u, u_t, u_{tt}) = (u_0, u_1, u_2) & \text{on } \{t = 0\} \times \Omega, \\ (u, \Delta u) = (g, h) & \text{on } \mathbb{R}_+ \times \Gamma, \end{cases}$$

which is motivated from the Blackstock–Crighton equation modeling the propagation of finite-amplitude sound in thermoviscous fluids. The spatial domain $\Omega \subset \mathbb{R}^n$ is assumed to have smooth boundary Γ. Moreover, $u_0, u_1, u_2 \colon \Omega \to \mathbb{R}$ and $g, h \colon J \times \Gamma \to \mathbb{R}$ are given, $u = u(t, x)$ is the unknown, and a, b, c, k are positive constants.

We show that, for small initial and boundary data, there exists a unique global solution with optimal L_p-regularity which converges to zero at an exponential rate as time tends to infinity. Our techniques are based on maximal L_p-regularity for parabolic problems and the implicit function theorem.

Additionally, we will provide a short outlook on how to treat the case of non-homogeneous Neumann boundary conditions which are relevant for applications of high intensity focused ultrasound in a medical context.

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