Long time behavior of the quadratic Klein-Gordon equation in the nonrelativistic limit regime

Yong Lu*†[1] and Zhifei Zhang‡[2]

[1]Mathematical Institute, Charles University in Prague – Sokolovska 83, 186 75 Prague 8, Czech Republic

[2]School of Mathematical Science, Peking University – 100871 Beijing, China

Abstract

We consider the following Cauchy problem of the Klein-Gordon equation

$$\begin{cases} \varepsilon^2 \partial_{tt} u - \Delta u + \frac{1}{\varepsilon^2} u + f(u) = 0, \quad t \geq 0, \quad x \in \mathbb{R}^d, \\ u(0) = u_{0,\varepsilon}, \quad (\partial_t u)(0) = \frac{1}{\varepsilon^2} u_{1,\varepsilon}. \end{cases} \tag{1}$$

Here $u = u(t,x)$ is a real-valued (or complex-valued) field, and $f(u)$ is a real-valued function (or $f(u) = g(|u|^2)u$ if u is complex-valued). The non-dimensional parameter ε is proportional to the inverse of the speed of light.

We study the asymptotic behavior of the Klein-Gordon equation in the nonrelativistic limit regime $\varepsilon \to 0$. By employing the techniques in geometric optics, we show that the solution of the quadratic Klein-Gordon equation can be approximately described by a linear Schr'odinger equation with an error of order $O(\varepsilon)$ over a long time interval of order $O(\varepsilon^{-1})$. With general nonlinearities, we show that the Klein-Gordon equation can be approximated by nonlinear Schrödinger equations over time of order $O(1)$.

*Speaker

†Corresponding author: luyong@karlin.mff.cuni.cz

‡Corresponding author: zfzhang@math.pku.edu.cn