Modelling medication: Phase resetting for a simple delay differential equation

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Abstract

Consider the equation

$$x'(t) = -\gamma x(t) + f(x(t-\tau))$$

for the density of a population of blood cells, say, neutrophils. Suppose a periodic solution corresponds to a disease, with local minima potentially dangerous for the patient. Medication is modelled by an increase of the production function f during a certain period of time. It is successful if the minimum after medication is larger than that of the periodic solution. We begin with the simplest case where f is a one-step function. This enables us to compute solutions explicitly. For suitable γ and f there is a stable and attracting periodic orbit. We calculate solutions in case of medication and find that the effect of medication depends on the phase in which the periodic solution is at the onset of medication. Among others we obtain phase intervals in which medication is useless. For a model with a two-step function, which should be a bit closer to reality, we obtain that medication when administered in the wrong phase even lowers the next minimum and so has an adverse effect.

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