Attractors for Damped Semilinear Wave Equations with a Robin–Acoustic Boundary Perturbation

Sergio Frigeri[1] and Joseph Shomberg*†[2]

[1]Weierstrass Institute for Applied Analysis and Stochastics (WIAS) – Germany
[2]Providence College (PC) – Providence College; 1 Cunningham Square; Providence, RI; 02918; USA, United States

Abstract

Under consideration is the damped semilinear wave equation

$$u_{tt} + u_t - \Delta u + u + f(u) = 0$$

on a bounded domain Ω in $\mathbb{R}^3$ with a perturbation parameter $\varepsilon > 0$ occurring in an acoustic boundary condition, limiting ($\varepsilon = 0$) to a Robin boundary condition.
With minimal assumptions on the nonlinear term f, the existence and uniqueness of global weak solutions is shown for each $\varepsilon \in [0, 1]$.
Also, the existence of a family of global attractors is shown to exist (re: J. Ball's generalized semiflows).
After proving a general result concerning the upper-semicontinuity of a one-parameter family of sets, the result is applied to the family of global attractors.
No further regularity from the global attractors is needed in order to obtain this upper-semicontinuity result.
With more relaxed assumptions on the nonlinear term f, we are able to show the global attractors possess optimal regularity and prove the existence of an exponential attractor, for each $\varepsilon \in [0, 1]$.
This result insures that the corresponding global attractor inherits finite (fractal) dimension; however, the dimension is *not* necessarily uniform in ε.

*Speaker
†Corresponding author: jshomber@providence.edu